

Interaction of the Electromagnetic p-Waves with Thin Metal Films

A. V. Latyshev¹ and A. A. Yushmanov²

*Faculty of Physics and Mathematics,
Moscow State Regional University, 105005,
Moscow, Radio st., 10-A*

Abstract

For the first time it is shown that for thin metallic films thickness of which not exceed thickness of skin – layer, the problem allows analytical solution for arbitrary boundary value problems. The analysis of dependence of coefficients of transmission, reflection and absorption on angle incidence, thickness of films and coefficient of specular reflection is carried out.

Key words: degenerate collisional plasma, electromagnetic p -wave, thin metallic film, coefficients of transmission, reflection and absorption.

PACS numbers: 73.50.-h Electronic transport phenomena in thin films, 73.50.Mx High-frequency effects; plasma effects, 73.61.-r Electrical properties of specific thin films, 73.63.-b Electronic transport in nanoscale materials and structures

1. Introduction

The problem of interaction of an electromagnetic wave with the metal films draws attention to itself for the long time [1] – [5]. It is connected as with theoretical interest to this problem, and with numerous practical appendices as well.

Researches of interaction of an electromagnetic wave with conducting medium (in particular, metal films) were carried out basically for a case of specular dissipation of electrons on a film surface. It is connected with the fact that for more general boundary conditions the problem becomes essentially complicated and does not suppose the analytical solution generally. At the same time for the real materials coefficient of specular reflection of

¹avlatyshev@mail.ru

²yushmanov@inbox.ru

electrons reflection from surfaces is far from unit as a rule. For example, in the work [6] on the basis of the analysis of longitudinal magnetic resistance of the thin metal wire it is shown, that for sodium coefficient of specular reflection is equal to 0.3.

In the present work it is shown that for thin films, a thickness of which does not exceed a thickness of a skin – layer, the problem allows the analytical solution for any boundary conditions.

Let's notice, that the most part of reasonings carrying out below is fair for more general case of conducting medium (in particular, semi-conductor) films.

PROBLEM STATEMENT

We consider the thin slab of conducting medium on which the electromagnetic wave falls. The angle of incidence we denote by θ . We will assume, that the vector of magnetic field of electromagnetic wave is parallel to the surface of slab. Such wave is called p -wave [3] or [1].

We take Cartesian coordinate system with origin of coordinates on one of the surfaces of a slab, with axes x , directed deep into the slab. We direct the axes y parallel with the vector of magnetic field of electromagnetic wave. At such choice of system of coordinates the electric field vector and magnetic field vector have the following structure

$$\mathbf{E} = \{E_x(x, y, z), 0, E_z(x, y, z)\} \text{ and } \mathbf{H} = \{0, H_y(x, y, z), 0\}.$$

Further components of electric and magnetic field vectors we search in the form

$$E_x(x, z, t) = E_x(x)e^{-i\omega t + ik \sin \theta z}, \quad E_z(x, z, t) = E_z(x)e^{-i\omega t + ik \sin \theta z},$$

and

$$H_y(x, z, t) = H_y(x)e^{-i\omega t + ik \sin \theta z}.$$

Now behaviour of electric and magnetic fields of a wave in the slab is

described by the following system of differential equations [3]

$$\begin{cases} \frac{dE_z}{dx} - ik \sin \theta E_x + ik H_y = 0 \\ ik E_x - ik \sin \theta H_y = \frac{4\pi}{c} j_x \\ \frac{dH_y}{dx} + ik E_z = \frac{4\pi}{c} j_z. \end{cases} \quad (1)$$

Here c is the velocity of light, \mathbf{j} is the density of current, k is the wave number.

We denote the thickness of slab by d .

The coefficients of transmission T , reflection R and absorption A of the electromagnetic wave by slab are described by following expressions [1], [7]

$$T = \frac{1}{4} |P^{(1)} - P^{(2)}|^2, \quad (2a)$$

$$R = \frac{1}{4} |P^{(1)} + P^{(2)}|^2, \quad (2b)$$

and

$$A = 1 - T - R. \quad (2c)$$

The quantities $P^{(j)}$ ($j = 1, 2$) are defined by following expressions

$$P^{(j)} = \frac{\cos \theta + Z^{(j)}}{\cos \theta - Z^{(j)}}, \quad j = 1, 2. \quad (3)$$

The quantities $Z^{(1)}$ and $Z^{(2)}$ correspond to impedance on the lower surface of slab by symmetrical configuration of the external magnetic field (this case 1 when

$$H_y(0) = H_y(d), \quad E_x(0) = E_x(d), \quad E_z(0) = -E_z(d))$$

and antysymmetrical configuration of the external magnetic field (this case 2 when

$$H_y(0) = -H_y(d), \quad E_x(0) = -E_x(d), \quad E_z(0) = E_z(d)).$$

The impedance is thus defined as follows

$$Z^{(j)} = \frac{E_z(-0)}{H_y(-0)}, \quad j = 1, 2. \quad (4)$$

We will consider the case when the width of the slab d is less than the depth of the skin – layer δ . Let's note, that depth of the skin – layer depends essentially on frequency of radiation, monotonously decreasing in process of growth of the last. The value δ possesses the minimum value in so-called infrared case [8]

$$\delta_0 = \frac{c}{\omega_p},$$

where ω_p is the plasma frequency.

For typical metals [8] $\delta_0 \sim 10^{-5}$ cm.

Hence for the films thickness of which d is less than δ_0 our assumption is fair for any frequencies.

The quantities H_y and E_z change a little on distances smaller than depth of a skin – layer.

Therefore under fulfilment of the given assumption $d < \delta$ these electrical and magnetical fields will change a little in the slab.

In case 1 when $H_y(0) = H_y(d)$ it is possible to accept that the value H_y is constant within the slab.

Variation of the quantity of y -projection of electric field by thickness of slab can be defined from the first equation of the system (1)

$$E_y(d) - E_y(0) = -ikdH_y + ik \sin \theta \int_0^d E_x dx. \quad (5)$$

From the second equation of the system (1) it follows that on the boundary of film the following relationship is satisfied

$$E_x(0) = E_x(d) = H_y \sin \theta. \quad (6)$$

The integral from relation (5) is proportional to the value of the quantity normal to the surface component of electrical field on the surface and therefore in according to the relation (6) to quantity H_y .

We define the coefficient of proportionality as

$$G = \frac{1}{E_x(0)d} \int_0^d E_x(x) dx = \frac{1}{H_y d \sin \theta} \int_0^d E_x(x) dx. \quad (7)$$

With the help of (7) we rewrite the relation (5) in the following form

$$E_z(d) - E_z(0) = (-ikd + ikGd \sin^2 \theta) H_y.$$

Considering antisymmetric character of the projection of electric field E_y in this case we receive

$$E_y(0) = ikd(1 - G \sin^2 \theta) \frac{H_y}{2}. \quad (8)$$

Therefore for the impedance we have

$$Z^{(1)} = \frac{ikd}{2}(1 - G \sin^2 \theta). \quad (9)$$

For the case 2 when $E_z(0) = E_z(d)$, it is possible to assume that z – projection of electric field E_z is constant in the slab.

Then the magnetic field change on the width of a slab can be defined from the third equation of the system (1)

$$H_y(d) - H_y(0) = -ikdE_z + \frac{4\pi}{c} \int_0^d j_z(x) dx. \quad (10)$$

Thus

$$j_z(x) = \sigma(x) E_z,$$

where $\sigma(x)$ is the conductance that in general case depends on coordinate x .

Let's introduce the longitudinal conductivity averaged by thickness of the slab,

$$\sigma_d = \frac{1}{E_z d} \int_0^d j_z(x) dx = \frac{1}{d} \int_0^d \sigma(x) dx. \quad (11)$$

Then the relation (10) can be rewritten in the following form

$$H_y(d) - H_y(0) = -ikdE_z + \frac{4\pi d\sigma_d}{c}E_z.$$

Considering symmetry of the magnetic field, from here we have

$$H_y(0) = \frac{1}{2}ikdE_z - \frac{2\pi d\sigma_d}{c}E_z.$$

For the impedance (4) we have

$$Z^{(2)} = \frac{2c}{ickd - 4\pi\sigma_d d}. \quad (12)$$

From here we receive expressions for the quantities $P^{(j)}$ ($j = 1, 2$)

$$P^{(1)} = \frac{2\cos\theta + ikd(1 - G\sin^2\theta)}{2\cos\theta - ikd(1 - G\sin^2\theta)}, \quad (13a)$$

$$P^{(2)} = \frac{(4\pi\sigma_d - ikc)d\cos\theta - 2c}{(4\pi\sigma_d - ikc)d\cos\theta + 2c}. \quad (13b)$$

We will assume that length of a wave of incident radiation surpasses essentially the thickness of the slab, i.e. $kd \ll 1$. Then expressions (9) and (12) for impedances and expression (13) for quantity $P^{(j)}$ ($j = 1, 2$) become a little simpler

$$Z_0^{(1)} = -\frac{1}{2}ikGd\sin^2\theta, \quad Z_0^{(2)} = -\frac{c}{2\pi\sigma_d d}. \quad (14)$$

Substituting (14) into (3), we have:

$$P_0^{(1)} = \frac{2\cos\theta - ikGd\sin^2\theta}{2\cos\theta + ikGd\sin^2\theta}, \quad P_0^{(2)} = \frac{2\pi\sigma_d d\cos\theta - c}{2\pi\sigma_d d\cos\theta + c}. \quad (15)$$

Thus quantities R, T, A can be found according to the formulas (2).

In a limiting case of non-conducting slab, when $\sigma_d \rightarrow 0$, $G \rightarrow 1$ from these expressions we have

$$P^{(1)} = -P^{(2)} = \frac{2 + ikd\cos\theta}{2 - ikd\cos\theta},$$

from which

$$T = 1, \quad R = 0, \quad A = 0.$$

Under almost tangent incidence when $\theta \rightarrow \pi/2$ we receive $P^{(1)} \rightarrow -1, P^{(2)} \rightarrow -1$. Thus, we obtain that $T \rightarrow 0, R \rightarrow 1, A \rightarrow 0$.

Let the relation $kl \ll 1$ be true. Then in a low-frequency case, when $\omega \rightarrow 0$, the quantity σ_d for a metal film can be presented in the following form [9]

$$\sigma_d = \frac{w}{\Phi(w)} \sigma_0, \quad w = \frac{d}{l}, \quad (16)$$

where

$$\frac{1}{\Phi(w)} = \frac{1}{w} - \frac{3}{2w^2}(1-p) \int_1^\infty \left(\frac{1}{t^3} - \frac{1}{t^5} \right) \frac{1 - e^{-wt}}{1 - pe^{-wt}} dt.$$

Here l is the mean free path of electrons, p is the coefficient of specular reflection, $\sigma_0 = \omega_p^2 \tau / (4\pi)$ is the static conductivity of volume pattern, $\tau = l/v_F$ is the time of mean free path of electrons, v_F is the Fermi's velocity.

In a low-frequency case when the formula (16) is applicable, the coefficients T, R, A according to formulas (2) do not depend on frequency of the incoming radiation.

For any frequencies these expressions are true under condition, that it is necessary to use the following expression $l \rightarrow \frac{v_F \tau}{1 - i\omega\tau}$, as a quantity l and the expression $\sigma_0 \rightarrow \frac{\sigma_0}{1 - i\omega\tau}$ instead of σ_0 .

For a case $kl \ll 1$ the quantity G can be calculated from the problem of behaviour of a plasma slab in variable electric field, which is perpendicular to surface of slab [10].

Let's calculate the coefficients of transmission and reflection in the case when $kd \ll 1$. We will substitute expressions $P_0^{(1)}$ and $P_0^{(2)}$, defined according to (15), into the formulas (2). We receive

$$T = \cos^2 \theta \left| \frac{1 - ik \frac{d}{2} G \sin^2 \theta \frac{2\pi d \sigma_d}{c}}{(\cos \theta + ik \frac{d}{2} G \sin^2 \theta) \left(1 + \frac{2\pi d \sigma_d}{c} \cos \theta \right)} \right|^2, \quad (17)$$

$$R = \left| \frac{ik \frac{d}{2} G \sin^2 \theta - \frac{2\pi d \sigma_d}{c} \cos^2 \theta}{(\cos \theta + ik \frac{d}{2} G \sin^2 \theta)(1 + \frac{2\pi d \sigma_d}{c} \cos \theta)} \right|^2, \quad (18)$$

Let's consider a case of a thin slab of sodium. Thus [1] $\omega_p = 6.5 \times 10^{15} \text{ sec}^{-1}$, $v_F = 8.52 \times 10^7 \text{ cm/sec}$. We consider a case of low frequencies $0 < \omega \leq 0.2\omega_p$. In this case the quantity $G < 1$, therefore $|kdG| \ll 1$. Formulas (17) and (18) become simpler essentially and have the following form

$$T = \frac{1}{\left| 1 + \frac{2\pi d \sigma_d}{c} \cos \theta \right|^2} \quad (19)$$

and

$$R = \left| \frac{\frac{2\pi d \sigma_d}{c} \cos \theta}{1 + \frac{2\pi d \sigma_d}{c} \cos \theta} \right|^2. \quad (20)$$

Using formulas (19) and (20), we will carry out graphic research of coefficients of transmission, reflection and absorption.

DISCUSSION OF RESULTS

Let's put, that $\tau\omega_p = 0.001$ [1]. Then $\tau = 1.5 \cdot 10^{-13} \text{ sec}$, i.e. $\nu = \frac{1}{\tau} = 0.667 \cdot 10^{13} \text{ sec}^{-1}$.

The coefficient of specular reflectivity we will take equal to 0.3 (Figs. 1–6).

We investigate dependence of coefficients of transmission, reflection and absorption on quantity of oscillations frequency of electromagnetic field ($0 < \omega \leq 0.2\omega_p$).

On figs 1 - 3 we will represent dependence of coefficients of transmission (fig.1), reflection (fig. 1) and absorption (fig. 3) on varying values of quantities of thickness of the slab in case of normal incidence of the electromagnetic wave ($\theta = 0$). The curves 1, 2, 3 correspond to the following values of thickness of slab $d = 10^{-6} \text{ cm}$, $0.9 \cdot 10^{-6} \text{ cm}$, $0.8 \cdot 10^{-6} \text{ cm}$.

Plots in these figures show, that in the whole range of change of frequency of electromagnetic wave oscillations $0 \leq \omega \leq 0.2\omega_p$ values of coeffi-

coefficients of transmission and absorption decrease with growth of a thickness of the slab, and values of coefficients of reflection increase.

Thus the absorption coefficients has maximum at $\omega \approx 10^{14} \text{ sec}^{-1}$.

On figs 4–6 we represent dependence of coefficients of transmission (fig. 4), reflection (fig. 5) and absorption (fig. 6) at various values quantities of angle of falling of an electromagnetic wave.

Graphics in these figures show that in all range of change of frequency of electromagnetic wave oscillations with growth of quantity an angle of incidence of the electromagnetic wave values of transmission and absorption increase, and values of reflection coefficient decreases. At small values of an angle of incidence change of the absorption coefficient have non-monotonic character.

On figs 7 – 9 dependence of coefficients of transmission (fig. 7), reflection (fig. 8) and absorption (fig. 9) on various values of coefficients of specular reflection is represented. Curves 1, 2, 3 correspond to values of coefficients of specular reflection $p = 0, 0.5, 0.8$. The thickness of the slab is equal to $d = 10^{-6} \text{ cm}$.

The analysis of these figures shows, that in whole range of change frequencies of oscillations of an electromagnetic wave values of transmission and absorption coefficients decrease, and values of coefficients of specular reflection increase with growth of coefficient of specular reflection. Under small values of coefficients of specular reflection change of absorption coefficients has non-monotonic character.

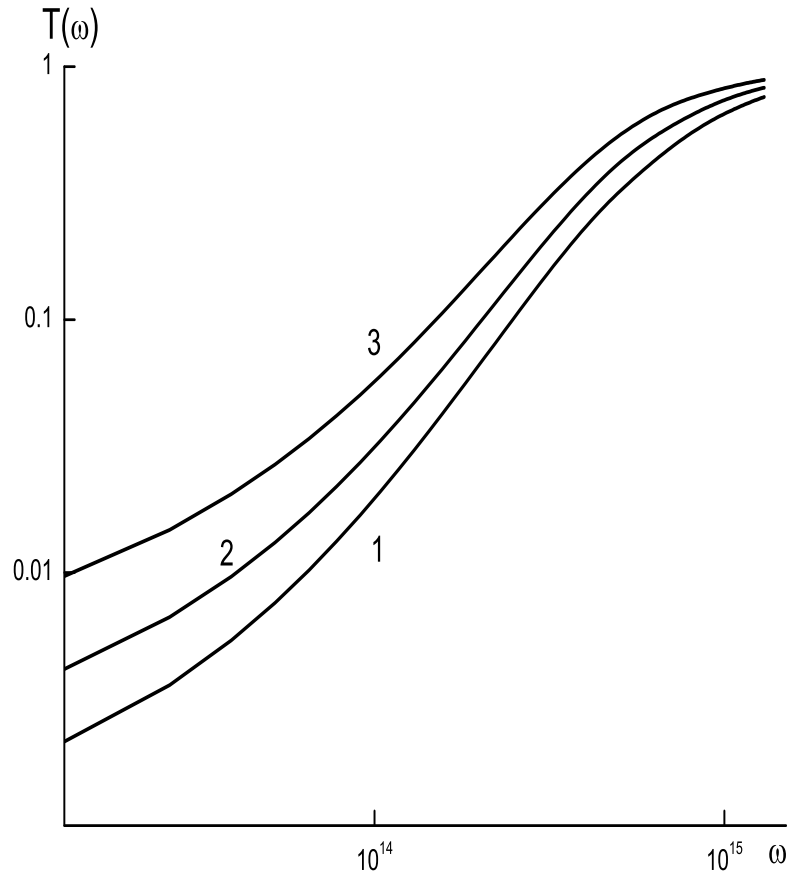


Figure 1. Transmission coefficient. Curves 1, 2, 3 correspond to quantities the thickness of slab $d = 10^{-6}$ cm, $0.9 \cdot 10^{-6}$ cm, $0.8 \cdot 10^{-6}$ cm.

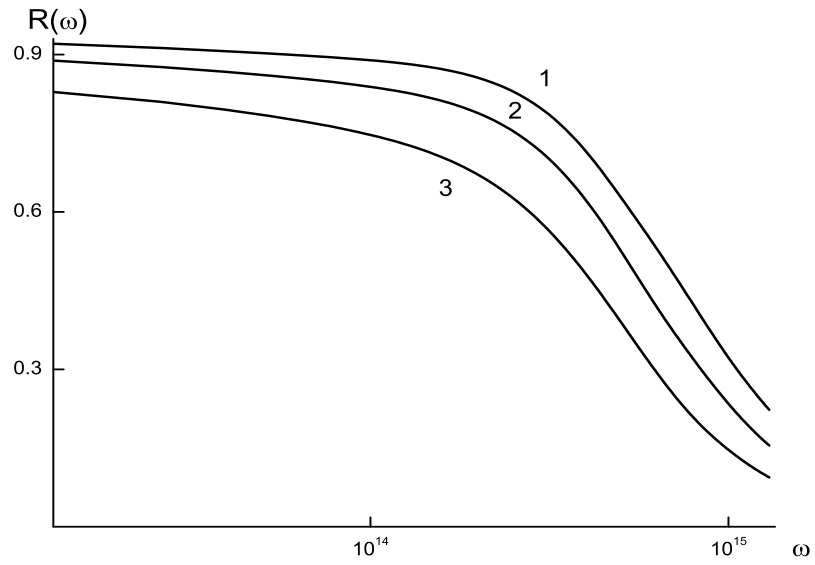


Figure 2. Reflection coefficient. Curves 1, 2, 3 correspond to quantities the thickness of slab $d = 10^{-6}$ cm, $0.9 \cdot 10^{-6}$ cm, $0.8 \cdot 10^{-6}$ cm.

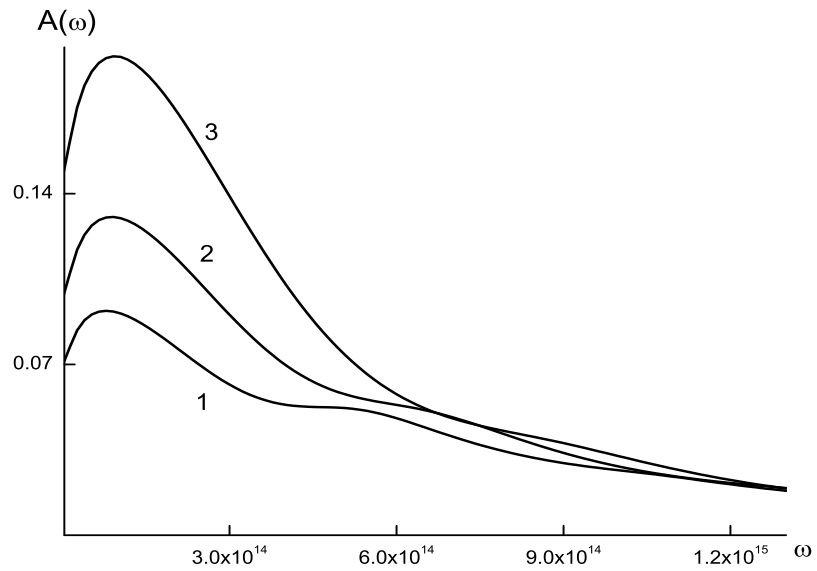


Figure 3. Absorption coefficient. Curves 1, 2, 3 correspond to quantities the thickness of slab $d = 10^{-6}$ cm, $0.9 \cdot 10^{-6}$ cm, $0.8 \cdot 10^{-6}$ cm.

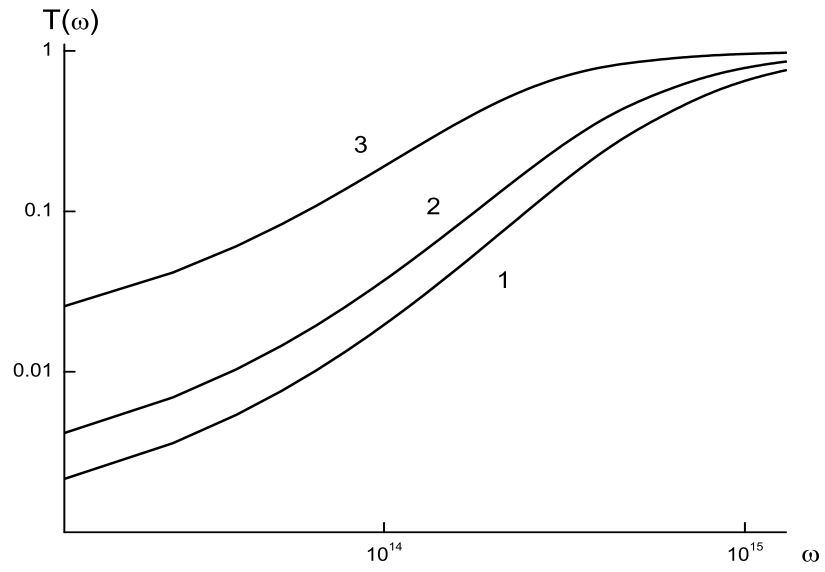


Figure 4. Transmission coefficient. Curves 1, 2, 3 correspond to quantities the angle of incidence $\theta = 0, \frac{\pi}{4}, \frac{5\pi}{12}$.

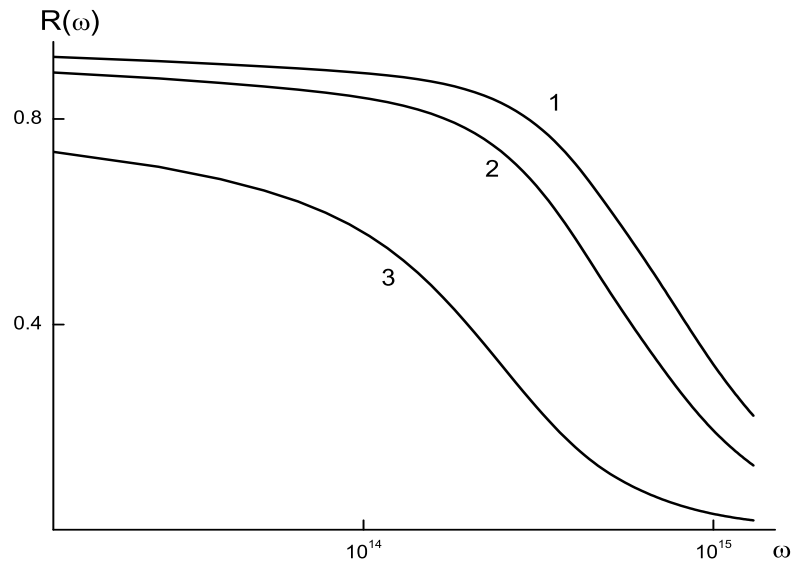


Figure 5. Reflection coefficient. Curves 1, 2, 3 correspond to quantities the angle of incidence $\theta = 0, \frac{\pi}{4}, \frac{5\pi}{12}$.

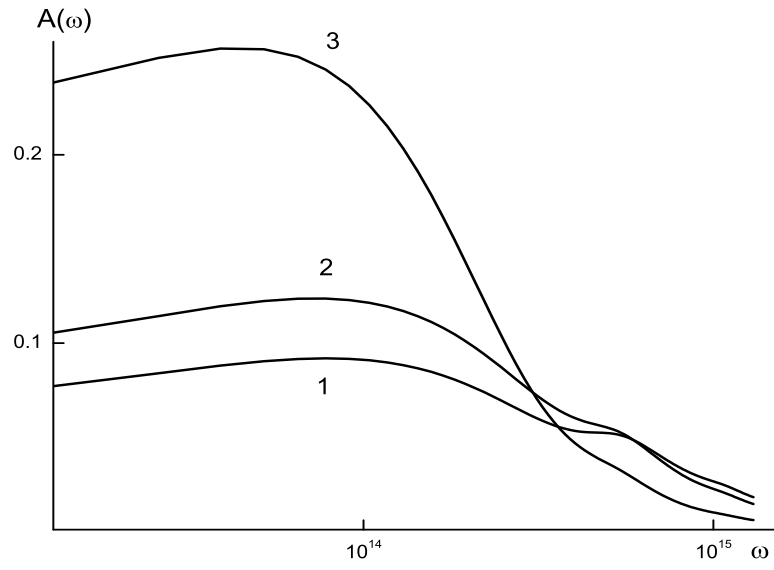


Figure 6. Absorption coefficient. Curves 1, 2, 3 correspond to quantities the angle of incidence $\theta = 0, \frac{\pi}{4}, \frac{5\pi}{12}$.

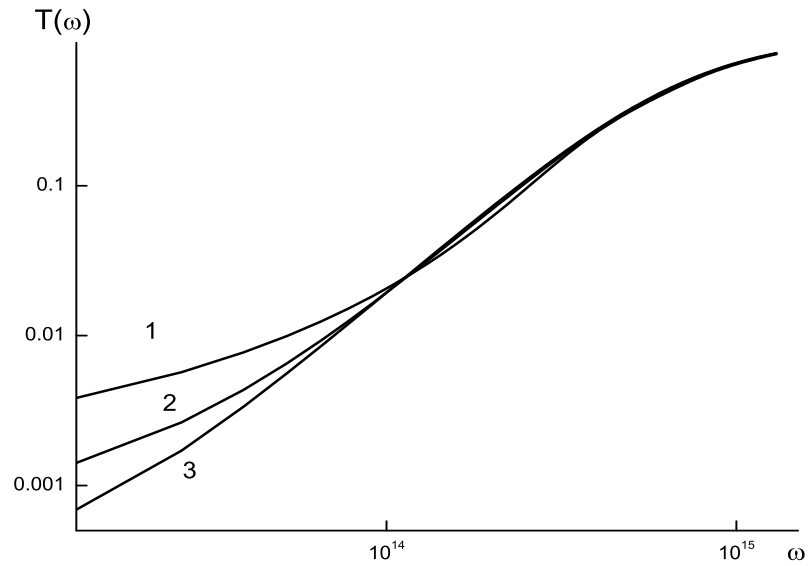


Figure 7. Transmission coefficient. Curves 1, 2, 3 correspond to quantities the coefficient of specular reflection $p = 0, 0.5, 0.8$.

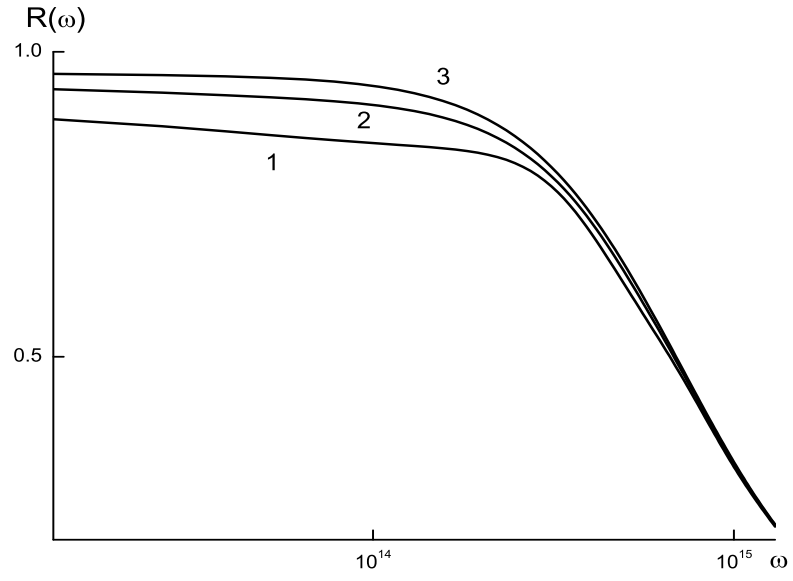


Figure 8. Reflection coefficient. Curves 1, 2, 3 correspond to quantities the coefficient of specular reflection $p = 0, 0.5, 0.8$.

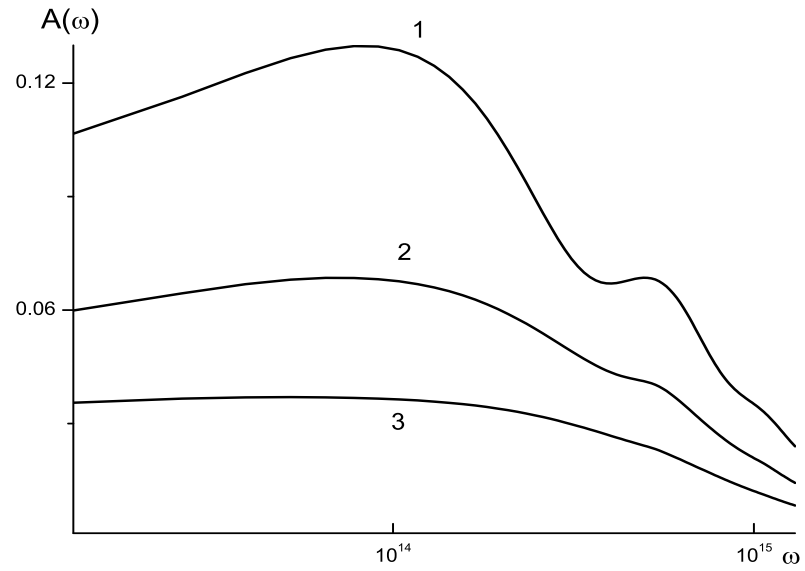


Figure 9. Absorption coefficient. Curves 1, 2, 3 correspond to quantities the coefficient of specular reflection $p = 0, 0.5, 0.8$.

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